ME 18b, HW 1

Due Tuesday April 7, 2008 (by 4 pm)

Suggested Solutions

1.1 Thermal efficiency for an Ideal Rankine cycle. (11.14)

Suppose you wish to design a solar powered Rankine cycle plant using water as a working fluid. If the solar collector exit has saturated water vapor at a pressure of 900 kPa and the condenser operates at 10 kPa determine the following: (a) the operating temperatures of the evaporator and the condenser. (b) The thermal efficiency of this ideal power plant. (c) Compare this to the thermal efficiency of a Carnot Cycle plant.

Solution:

Water ideal Rankine cycle, see diagram below. Using CATT 3 for thermodynamic properties:

State 3: Sat vapor at P3 = 900 kPa ⇒ T3 = 175°C, h3 = 2774 kJ/kg, s3 = 6.623 kJ/kg K

State 1: Sat liquid at P1 = 10 kPa ⇒ T1 = T4 = 46°C, v1 = 0.00101 m3/kg, h1 = 192 kJ/kg

(a) Evaporator Temp = 175°C, Condenser Temp = 46°C

Process: Turbine is adiabatic and reversible so second law gives

s4 = s3 = 6.623 kJ/kg K and P4 = 10 kPa => h4 = 2097 kJ/kg

The energy equation gives

wT = h3 - h4 = 2774 - 2097 = 677 kJ/kg

Process: Pump and incompressible liquid gives work into pump

wP = v1(P2 - P1) = 0.00101(900 - 10) = 0.90 kJ/kg

h2 = h1 + wP = 192 + 0.90 = 193 kJ/kg

Process: Boiler gives the heat transfer from the energy equation as

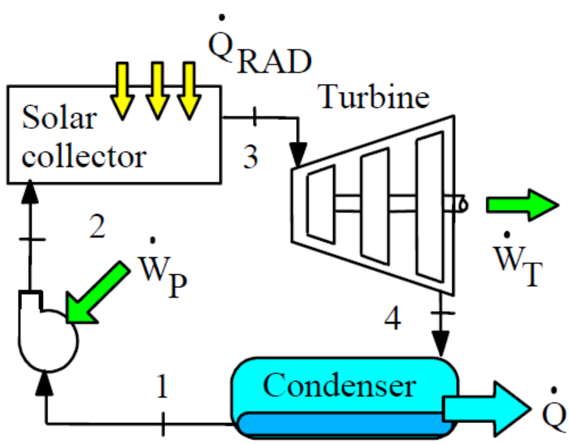
qH = h3 - h2 = 2774 - 193 = 2581 kJ/kg

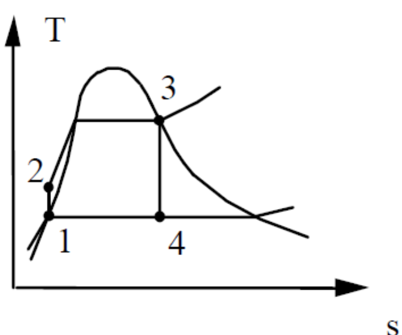
The cycle net work and efficiency are found as

wNET = wT - wP = 677 - 0.9 = 676 kJ/kg

(b) **ηTH = wNET/qH = 676/2581 = 0.26**

(c) **ηcarnot = 1 – T_L/T_H = 1 – 319K/448K = 0.29**





1.2 Rankine cycle for power. (11.18)

A steam power plant has a high pressure of 3 MPa and it maintains 60°C in the condenser. The turbine exhaust quality must remain above 90%. Find the specific work and heat transfer in all components and the cycle efficiency. How does the specific work and heat transfer change if the boiler pressure was raised to 3.5 MPa?

Solution:

Based on the standard Rankine cycle and Table B.1. (alternately use CATT 3 software for properties)

State 1: Sat. liquid. $P_1 = 19.94$ kPa, $h_1 = 251.11$ kJ/kg, $v_1 = 0.001017$ m3/kg

Process: pump

Energy: $h_2 - h_1 = w_p = v_1 (P_2 - P_1) = 0.001017 (3000 – 19.94) =$ **3.03 kJ/kg**

State 2: $P_2 = 3000$ kPa, $h_2 = h_1 + w_p = 251.11 + 3.03 = 254.1$ kJ/kg

Process: Turbine

State 4: $P_4 = P_1 = 19.94$ kPa, $x = 0.9$

$s_4 = s_f + x_4 s_{fg} = 0.8311 + 0.9 \times 7.0784 = 7.20166$ kJ/kg-K

$h_4 = h_f + x_4 h_{fg} = 251.11 + 0.9 \times 2358.48 = 2373.74$ kJ/kg

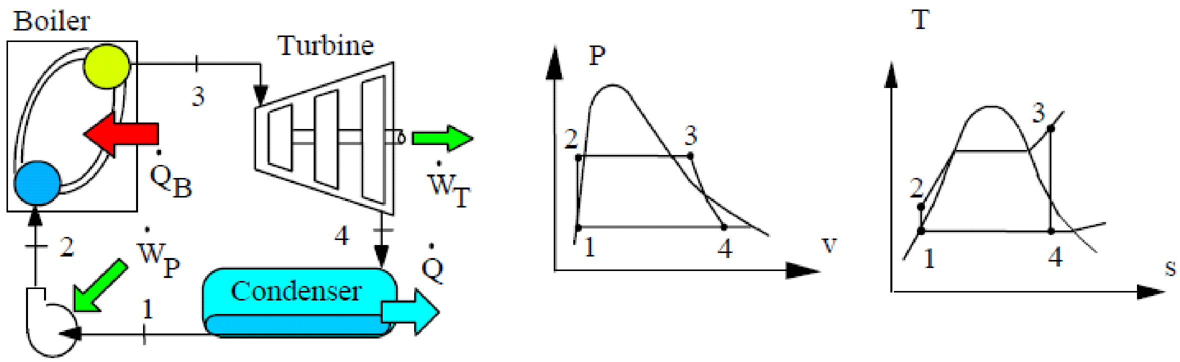
State 3: Table B.2.2 3000 kPa, $s_3 = 7.20166$ kJ/kg K => $h_3 = 3432.5$ kJ/kg

Boiler: $q_H = h_3 – h_2 = 3432.5 – 254.1 =$ **3178.4 kJ/kg**

Turbine: $w_T = h_3 – h_4 = 3432.5 – 2373.74 =$ **1058.8 kJ/kg**

Condenser: $q_L = h_4 – h_1 = 2373.74 – 251.1 =$ **2122.6 kJ/kg**

Efficiency: $\eta_{TH} = w_{NET}/q_H = (w_T - w_P)/q_H = (1058.8 - 3.03) / 3178.4 =$ **0.332**



For boiler at 3.5 MPa:

State 1: Sat. liquid. $P_1 = 19.94$ kPa, $h_1 = 251.11$ kJ/kg, $v_1 = 0.001017$ m3/kg

Process: pump

Energy: $h_2 - h_1 = w_p = v_1 (P_2 - P_1) = 0.001017 (3500 – 19.94) =$ **3.54 kJ/kg**

State 2: $P_2 = 3500$ kPa, $h_2 = h_1 + w_p = 251.11 + 3.54 = 254.6$ kJ/kg

Process: turbine

State 4: $P_4 = P_1 = 19.94$ kPa, $x = 0.9$

$s_4 = s_f + x_4 s_{fg} = 0.8311 + 0.9 \times 7.0784 = 7.20166$ kJ/kg-K

$h_4 = h_f + x_4 h_{fg} = 251.11 + 0.9 \times 2358.48 = 2373.74$ kJ/kg

State 3: Table B.2.2 3500 kPa, $s_3 = 7.20166$ kJ/kg K => $h_3 = 3486$ kJ/kg

Boiler: $q_H = h_3 – h_2 = 3486 – 254.6 =$ **3231.4 kJ/kg**

Turbine: $w_T = h_3 – h_4 = 3486 – 2373.7 =$ **1112.3 kJ/kg**

Condenser: $q_L = h_4 – h_1 = 2373.74 – 251.1 =$ **2122.6 kJ/kg**

Efficiency: $\eta_{TH} = w_{NET}/q_H = (w_T - w_P)/q_H = (1112.3 - 3.54) / 3231.4 =$ **0.343**

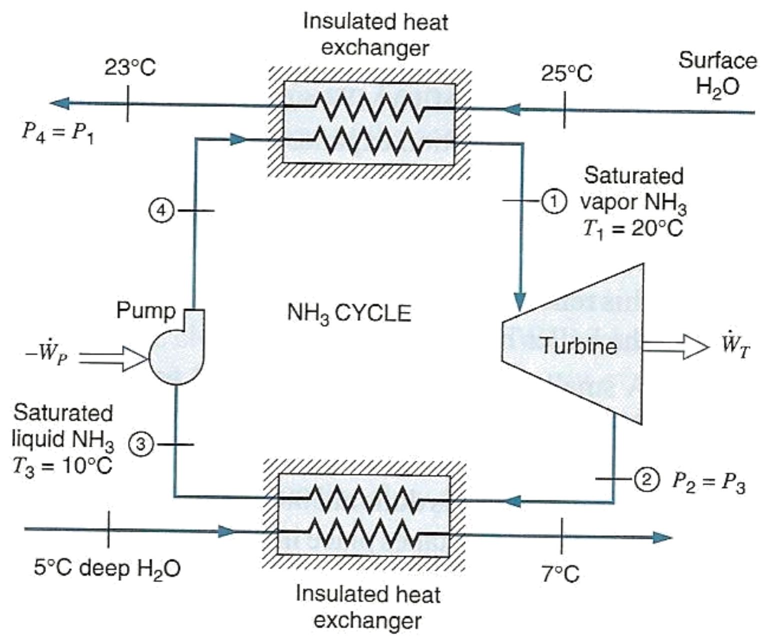
Summary:

Turbine output increases by 1112.3 – 1058.8 = 53.5 kJ/kg
Heat Input increases by 3231.4 – 3178.4 = 53 kJ/kg
Pump input increases 0.51 kJ/kg
Cycle efficiency increases from 33.2% to 34.3%

1.3 Ocean Powered Rankine Plant (11.27)

The ammonia based Rankine-cycle power plant shown below was designed to operate in an ocean environment that is 25°C at the surface and 5°C at depth. If the ammonia mass flow rate is 1000 kg/s determine the following:

a. The turbine power output and the pump power input.
b. The mass flowrate of seawater through each heat exchanger
c. The thermal efficiency of the power plant.



Solution:

a) C.V. Turbine. Assume reversible and adiabatic.

s2 = s1 = 5.0863 = 0.8779 + x2 × 4.3269 => x2 = 0.9726

h2 = 227.08 + 0.9726 × 1225.09 = 1418.6 kJ/kg

w_T = h1 - h2 = 1460.29 - 1418.6 = 41.69 kJ/kg

Work Turbine = $\dot{m} \cdot w_T$ = 1000 × 41.69 = **41 690 kW**

Pump: $w_P \approx$ v3(P4 - P3) = 0.0016(857 - 615) = 0.387 kJ/kg

Work Pump = $\dot{m} \cdot w_P$ = 1000 × 0.387 = **387 kW**

b) Consider condenser heat transfer to the low T water

$\dot{Q}_{\text{to low T H2O}}$ = 1000(1418.6 - 227.08) = 1.1915×106 kW

$\dot{m}_{\text{low T H2O}}$ = 1.1915×10629.38 - 20.98 = **141 850 kg/s**

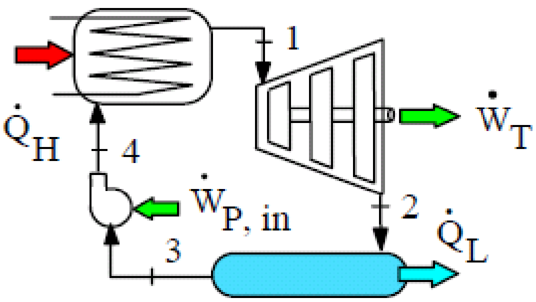
h4 = h3 + w_P = 227.08 + 0.39 = 227.47 kJ/kg

Now consider the boiler heat transfer from the high T water

$\dot{Q}_{\text{from high T H2O}}$ = 1000(1460.29 - 227.47) = 1.2328×106 kW

$\dot{m}_{\text{high T H2O}}$ = 1.2328×106104.87 – 96.50 = **147 290 kg/s**

c) $\eta_{TH} = \dot{W}_{NET}/\dot{Q}_H$ = 41 690 - 3871.2328×106 = **0.033**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$\dot{Q}_H$
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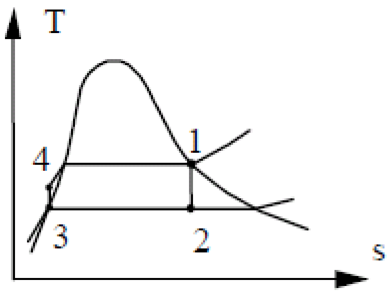
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