ME 18b, HW 3

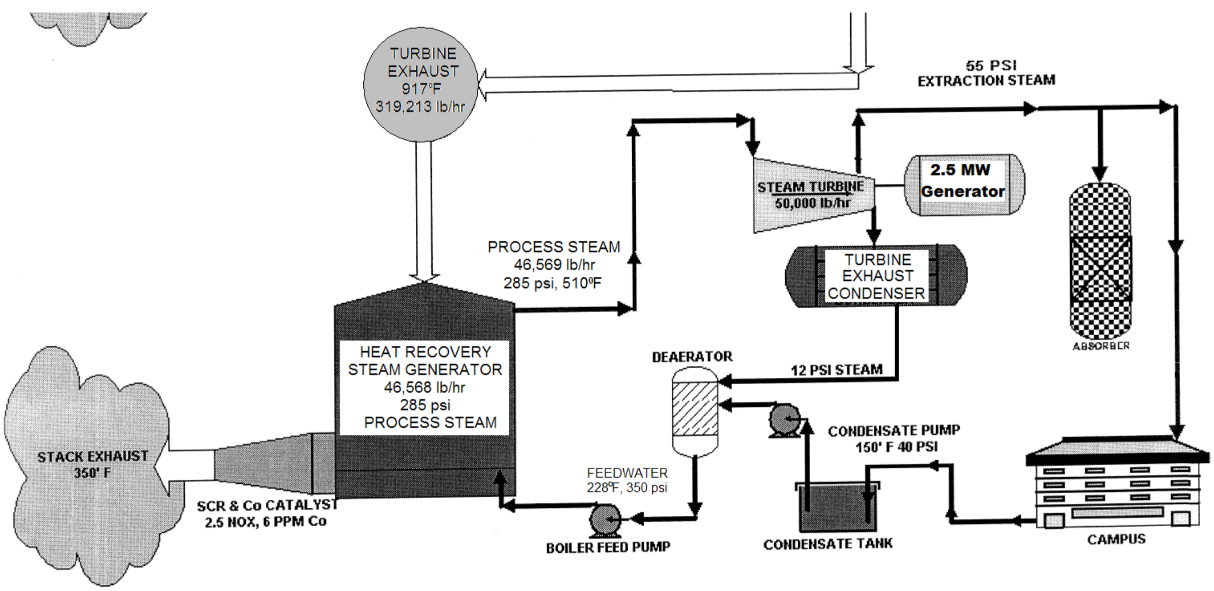
Due Tuesday April 21, 2008 (accepted until 4 pm)

TAs for HW #3:
Dan Alvarez, Monday 8-10 pm, SFL 229;
Mary Dorman, Monday 7-9 pm SFL 331

1. Cal Tech Power Plant Rankine Cycle analysis: The Cal Tech Power Plant uses two thermodynamic cycles to produce electric power and provide steam for campus use. Examine the following schematic of the power plant's Rankine cycle. (A better copy of this is posted as a handout on the class website). Using the information given on the diagram you should do a complete thermodynamic analysis of the cycle which includes finding:

[Note: Please convert all given values to SI units]

- **a)** The heat input of the steam generator (compare the heat lost by the gas turbine exhaust to the heat gained by the water)
- **b)** The work produced by the steam turbine, (compare this to the generated electric power of 2.5 MW – how efficient is the turbine/generator system)
- **c)** The work of the pump
- **d)** The overall Rankine cycle efficiency
- **e)** Suggest some ways the plant could become even more efficient



Solution:

Determine properties for each state in the cycle.

State 1, Pump exit: T1 = 109°C, P1 = 2.41 MPa, mass flow rate = 5.87 kg/s. Using CATT software find h1 = 459 kJ/kg

State 2, Steam Generator exit: T2 = 266°C, P2 = 1.97 MPa. Find h2 = 2944 kJ/kg, also s2 = 6.629 kJ/kgK. We'll need this to evaluate the efficiency of the turbine by comparing the actual output to the ideal turbine.

The steam generator heat input is: Q_{boiler} = m(h2 – h1) = 5.87 kg/s(2944 – 459)kJ/kg/1000

Q_{boiler} = 14.6 MW

Heat loss by gas turbine exhaust in the heat recovery unit. Assume the exhaust gas has properties similar to air. You could assume it's mostly a mixture of CO2 and H2O vapor but that makes it more complicated.

$Q_{exhaust} = m_{air}(h_{in} - h_{out})$

At T_{in} = 492°C, h_{in} = 784 kJ/kg, at T_{out} = 177°C, h_{out} = 452 kJ/kg

$Q_{exhaust}$ = 40.2 kg/s(784 – 452)kJ/kg/1000 = **13.3 MW**

It appears there isn't quite enough energy in the exhaust stream of the gas turbine to heat the steam. Additional energy on the order of 1.3 MW may be provided in the steam generator.

State 3, Turbine exit. For ideal turbine, s3 = s2. The back pressure, P3 = 0.083 MPa. Find that T3 = 94.5°C and h3 = 2376 kJ/kg.

The work output of the ideal turbine is:

W_{turb} = m(h2 – h3) = 5.87(2944 – 2376)/1000 = **3.3 MW**

The output of the generator is 2.5 MW so the efficiency of the turbine and generator combined is: **η_{turb}** = W_{actual}/W_{ideal} = 2.5/3.3 = **76%**

Neglecting the inefficiency of the generator, determine the actual state 3 conditions:

h3 = h2 - W_{turb}/m = 2944kJ/kg – (2500kW/5.87kg/s) = 2518 kJ/kg

We can find that steam quality at turbine exit is x3 = 93%

Work of the pump: W_{pump} = mv1(P2 – P1) = 5.87kg/s*0.001m^3/kg(2410 – 83)kPa

W_{pump} = 13.7 kW

Overall cycle efficiency: η_{th} = Wnet/Qin = (2500 – 13)kW/14,400kW

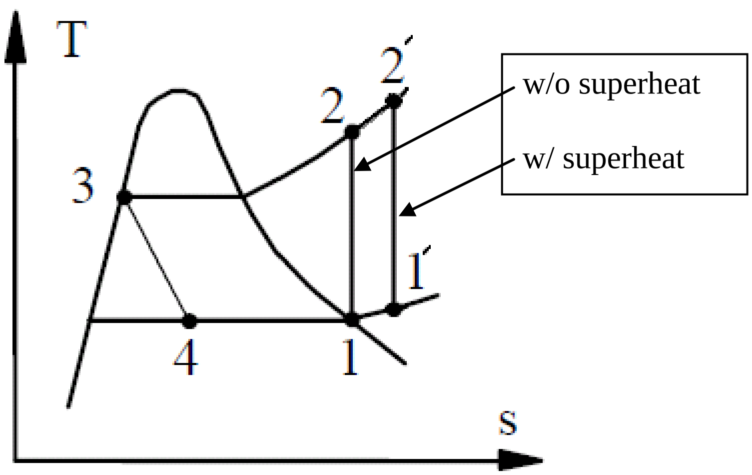
η_{th} = 17%

Which isn't really very high compared to typical utility power plants.

Things to help improve efficiency: superheat the vapor to a higher temperature, add steam reheat and use high pressure and low pressure turbines, use feedwater heaters. Although there probably isn't room for all this additional equipment in the plant. Cost is a major factor why the additional equipment isn't installed.

2. Refrigeration Cycle: A refrigeration cycle using R-134a has an evaporator exit temperature that is superheated by 8°C at a pressure of 600 kPa. The compressor has a pressure ratio of 2 and the refrigerant leaving the condenser is a saturated liquid. Compare the coefficient of performance, the compressor work per unit mass, the refrigerant temperature at the compressor exit for this cycle with those of an ideal cycle having no evaporator superheat. Draw the process for both cycles on a T-s diagram.

Solution:



Compressor Pressure ratio of 2 means the outlet pressure is 2x the inlet pressure.

Analysis: Find the thermodynamic properties at each state

No superheat case

State 1, compressor inlet (evaporator exit): At P1 = 600 kPa, $T1_{sat}$ = 21.6C, $h1_{sat}$ = 262 kJ/kg and $s1_{sat}$ = 0.920 kJ/kgK

State 2, compressor outlet: for P2 = 1200 kPa, s2 = 0.920 kJ/kgK, find that **T2 = 48.7C** and h2 = 276 kJ/kg.

Compressor work: w_{comp} = h1 – h2 = 262 – 276 = **-14 kJ/kg**

State 3: For P3 = 1200 kPa, sat. liq. Find h3 = 117 kJ/kg and T3 = 46.3C. These values are the same with or without superheat at the evaporator exit.

State 4: h4 = h3 (isenthalpic process in TXV)

Coefficient of Performance: $\beta r = -q_{41}/w_{12}$ = -(h1 – h4)/(h1 – h2)

βr = -(262 – 117)/(262 – 276) = **10.1**

With Superheat at evaporator exit

State 1: T1 = 29.6C, P1 = 600 kPa, find h1 = 270 kJ/kg, s1 = 0.947 kJ/kgK

State 2: for P2 = 1200 kPa, s2 = 0.947 kJ/kgK, find that **T2 = 56.2C**, h2 = 285 kJ/kg

Compressor work: w_{comp} = h1 – h2 = 270 – 285 = **-15 kJ/kg**

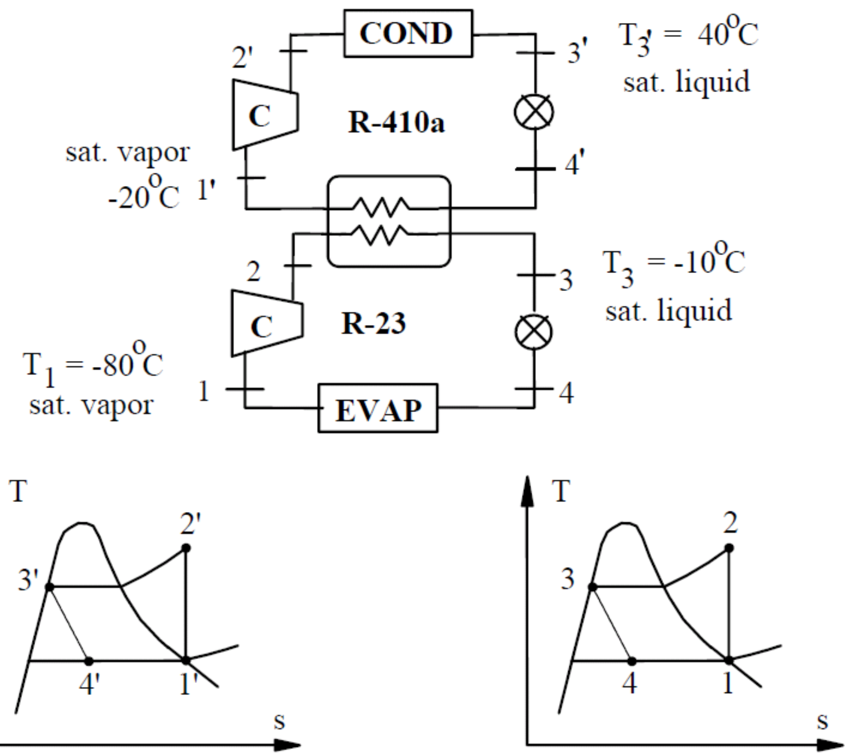
Coefficient of Performance: βr = $-q_{41}/w_{12}$ = -(h1 – h4)/(h1 – h2)

βr = -(270 – 117)/(270 – 285) = **10.2**

Comments: Superheated vapor in the compressor increases the work load on the compressor and only slightly increases the coefficient of performance due to increased heat absorption in the evaporator.

3. **Cascade Refrigeration Cycle:** A cascade system is composed of two ideal refrigeration cycles as shown below. The high-temperature cycle uses R-410a. Saturated liquid leaves the condenser at 40°C, and saturated vapor leaves the heat exchanger at -20°C. The low-temperature cycle uses R-23. Saturated vapor leaves the evaporator at -80°C with h = 330 kJ/kg, and saturated liquid leaves the heat exchanger at -10°C with h = 185 kJ/kg. R-23 out of the compressor has h = 405 kJ/kg. Calculate the ratio of the mass flow rates through the two cycles and the COP of the total system. Draw the cycle processes on a T-s diagram.

Solution:



All the enthalpy values for the bottom cycle are given, h1 = 330, h2 = 405, h3 = h4 = 185 kJ/kg so we just need to find the enthalpy values for the top cycle.

State 1': for sat. vap. At -20C, find h1' = 272 kJ/kg, s1' = 1.078 kJ/kgK

State 2': need to find pressure at state 3 first.

State 3': for sat. liq. At 40C, find h3' = h4' = 124 kJ/kg, P3' = P2' = 2.421 MPa

State 2': for P2' = 2.421 MPa and s2' = s1' = 1.078 kJ/kgK, find h2' = 323 kJ/kg

In the heat exchanger the heat lost by the bottom cycle condenser equals the heat gained by the top cycle evaporator:

$$Q = m' (h1' – h4') = m(h2 – h3)$$

The ratio of mass flow rates is given by:

$$m/m' = (h1' – h4')/ (h2 – h3)$$

$$\mathbf{m/m' =} (272 – 124)/(405 – 185) \mathbf{= 0.67}$$

The coefficient of performance is given by:

$$\beta_r = -Q_L/W_{tot} = -m \cdot q_L/ m \cdot w_{tot}$$

$$Q_L/m = (h1 – h4) = (330 – 185) = 145 \text{ kJ/kg}$$

$$Wtot = m(h2 – h1) + m'(h2' – h1')$$

$$-Wtot/m = (405 – 303) + (1/0.67)(323 – 272) = 150 \text{ kJ/kg}$$

$$\boldsymbol{\beta_r} \mathbf{=} 145/150 = \mathbf{0.96}$$

Comments It should be observed that refrigeration systems that pump heat over a large temperature difference don’t have a very high COP. In this problem we see that it takes about 1 watt of energy in the compressors to move 1 watt of heat from the low temperature evaporator to the high temperature condenser.

Solution using R-134a:

State 1': for sat. vap. At -20C, find h1' = 387 kJ/kg, s1' = 1.741 kJ/kgK

State 2': need to find pressure at state 3 first.

State 3': for sat. liq. At 40C, find h3' = h4' = 246 kJ/kg, P3' = P2' = 1.02 MPa

State 2': for P2' = 1.02 MPa and s2' = s1' = 1.741 kJ/kgK, find h2' = 429 kJ/kg

m/m' = (387 – 256)/(405 – 185) **= 0.60**

-Wtot/m = (405 – 303) + (1/0.60)(429 – 387) = 127 kJ/kg

$\boldsymbol{\beta_r}$ **=** 145/127 = **1.14**

Comments: It’s interesting to see that the choice of refrigerant affects the amount of compressor work required. In this case R-134a is a more energy efficient refrigerant.