

ME 18b, HW 4

Due Tuesday April 28, 2008 (accepted until 4 pm)

TAs for HW #4:

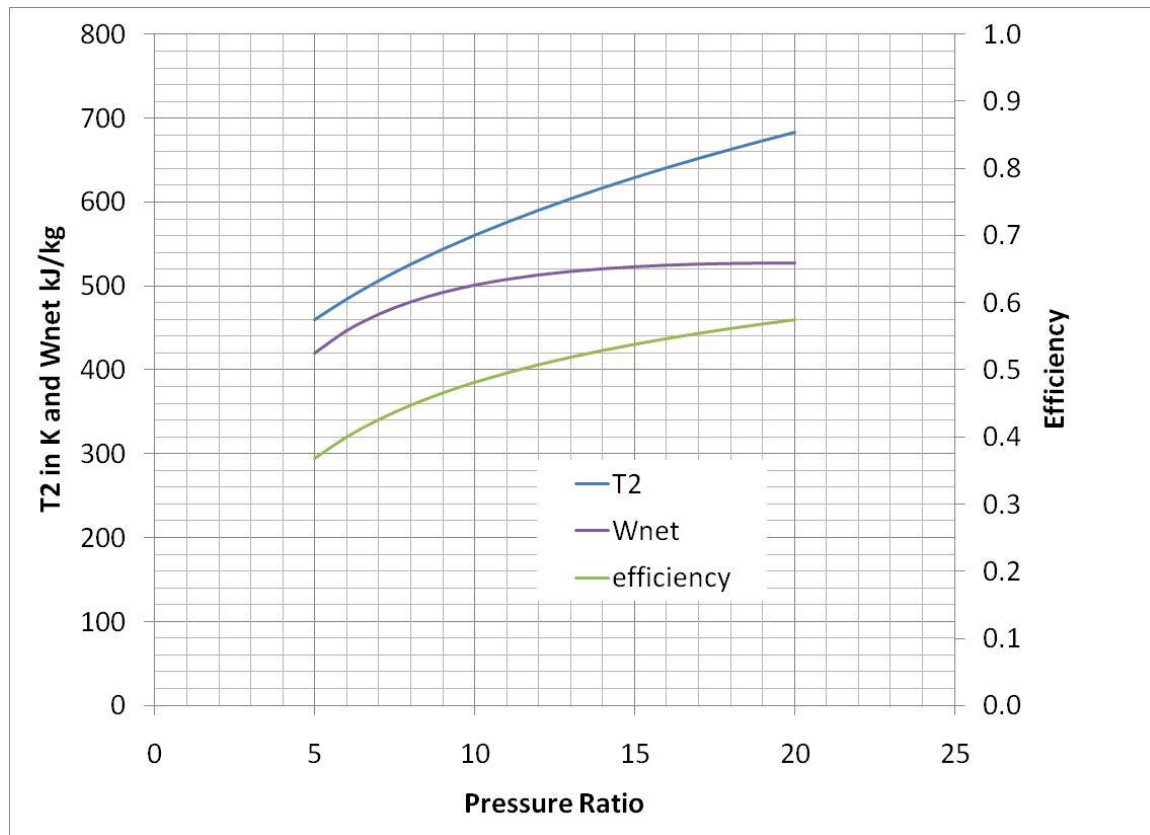
Xaiobai Li, Friday and Monday 3-4 pm, Tom 212E;

Julianne Gould, Sunday 4-6 pm, SFL 218

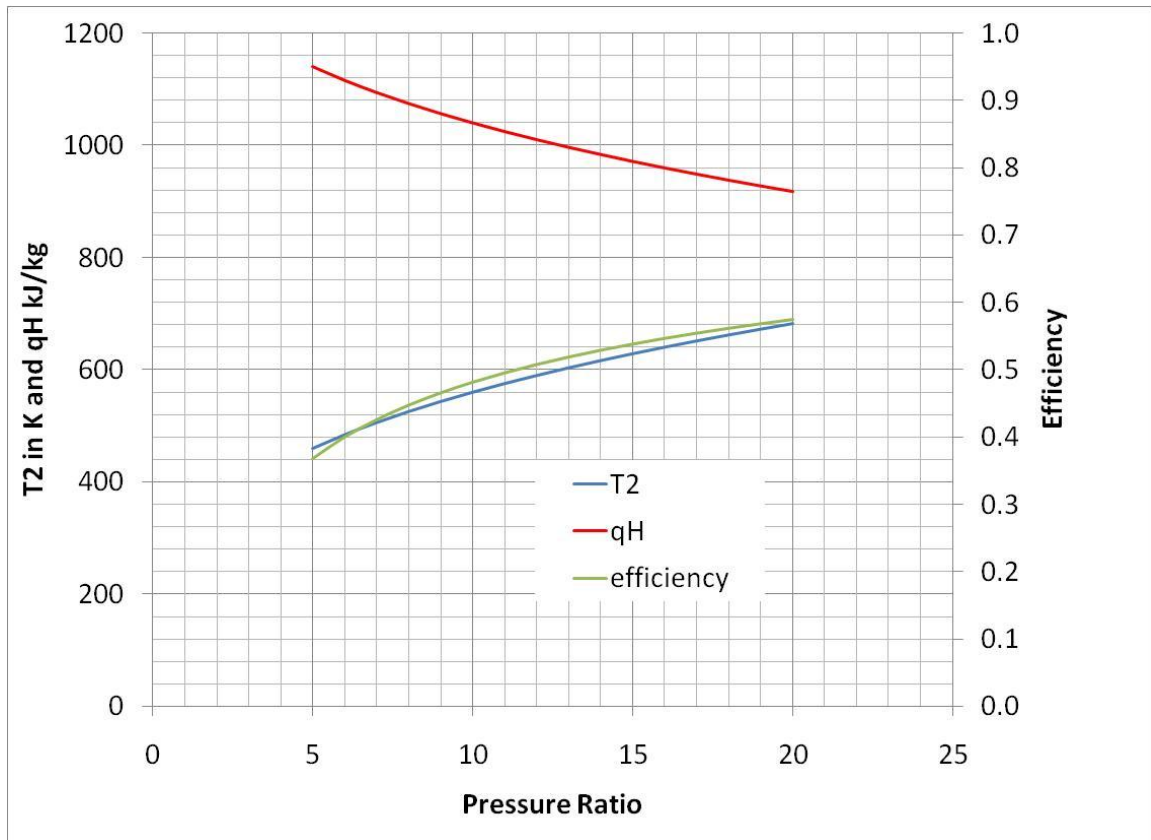
1. **Research the Brayton Cycle:** Obviously the Brayton cycle is named after the person who invented it, yet your textbook doesn't even mention this person's first name! Do a little research on your own about the Brayton cycle, briefly describe how it works, who invented it and tell a little bit of information about this inventor. Try to put your write up in your own words rather than cutting and pasting from the internet.

2. **Brayton Cycle based on Cold Air Standard:** Make plots of the specific heat transfer (into the cycle), efficiency and the specific work output for a Brayton cycle using cold air properties for a compression ratio ranging from 5 to 20. Assume the air inlet conditions are 290 K and 100 kPa and the turbine inlet temperature is 1600 K. Comment on the conclusions you observe in your plots. You may use a spreadsheet for this problem.

Solution: The plot below shows the compressor outlet temperature T_2 , efficiency of the cycle and the net work from the cycle for a pressure ratio from 5 to 20



The following plot shows the same results except for the heat input is shown with the cycle efficiency and the compressor outlet temperature T_2 .

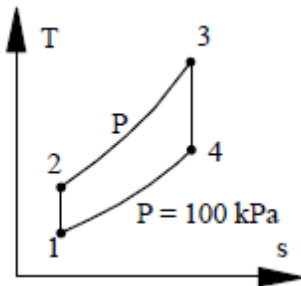


Comments: For a fixed high temperature limit (as would normally be the case in a real system) the required heat input decreases as the compression ratio increases. We note that the net work approaches a limit, in that it doesn't keep increasing. The efficiency increases because less heat has to be added to the cycle for about the same net work at higher compression ratios.

3. **Gas Turbine Cycle:** A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the maximum temperature is 1600 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1.
- Draw the process on a T-s diagram
 - Calculate the power output of the turbine.
 - What fraction of the turbine output is required to drive the compressor?
 - Calculate the mass flow rate of the gas in the cycle
 - What is the thermal efficiency of the cycle?

Solution:

The cycle T-s diagram is shown below.



Compressor is isentropic: $s_2 = s_1$

$$T_2 = T_1(P_2/P_1)^{(k-1)/k} = 300(14)^{0.286} = 638 \text{ K}$$

$$w_C = h_2 - h_1 = C_{p0}(T_2 - T_1) = 1.0 (638 - 300) = 338 \text{ kJ/kg}$$

Isentropic expansion in turbine: $s_4 = s_3$

$$T_4 = T_3 (P_4/P_3)^{(k-1)/k} = 1600 (1/14)^{0.286} = 752 \text{ K}$$

The specific power output of the turbine is given by $w_T = h_3 - h_4 = C_{p0}(T_3 - T_4)$

$$w_T = 1.0 (1600 - 752) = 848 \text{ kJ/kg}$$

$$w_{NET} = 848 - 338 = 510 \text{ kJ/kg}$$

Fraction of turbine power required to turn the compressor:

$$w_C/w_T = 338/848 = \mathbf{0.40}$$

$$m = W_{NET}/w_{NET} = 100,000 \text{ kW}/510 \text{ kJ/kg} = 196 \text{ kg/s}$$

The total power output of the turbine is:

$$W_T = m \cdot w_T = 196 \text{ kg/s} \times 848 = \mathbf{166 \text{ MW}}$$

The cycle efficiency is given by: $\eta_{TH} = w_{NET}/q_H$

Energy input to the combustor

$$q_H = C_{P0}(T_3 - T_2) = 1.0 (1600 - 638) = 962 \text{ kJ/kg}$$

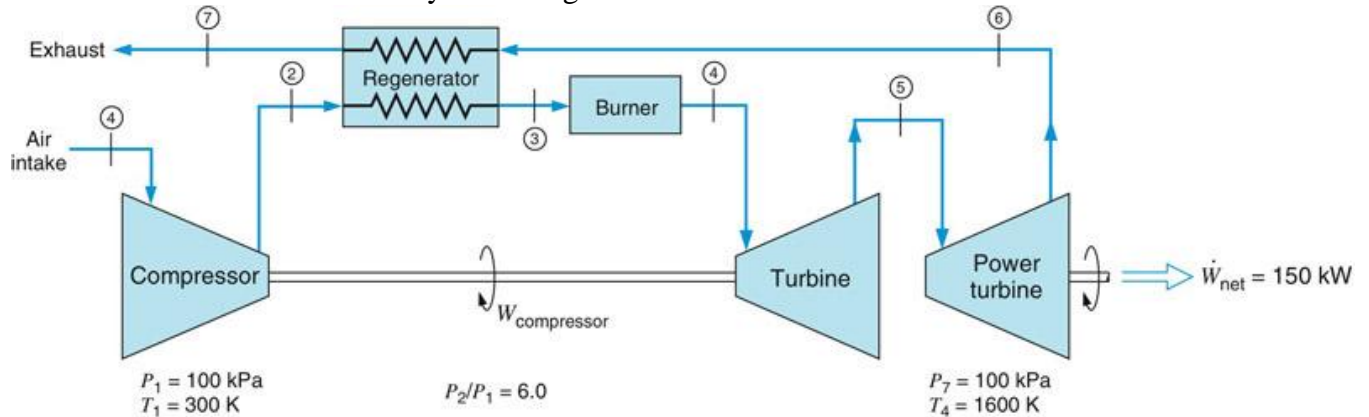
$$\eta_{TH} = 510/962 = \mathbf{0.53}$$

Comments: In gas cycles, the compressor work is a significant fraction of the total turbine work.

4. **Gas Turbine with Regenerator:** The gas turbine shown below is used as an automotive engine. In the first turbine, the gas expands to pressure P_5 , just low enough for this turbine to drive the compressor. The gas is then expanded through the second turbine connected to the drive wheels. The data for the engine are shown below. Assume all processes are ideal.

Determine the following:

- Intermediate pressure P_5
- Net specific work output
- Mass flow rate through the engine
- Air temperature entering the burner T_3
- Thermal efficiency of the engine



Solution:

(a) The compressor is an isentropic process: $s_2 = s_1 \Rightarrow T_2 = T_1(P_2/P_1)^{(k-1)/k}$

$$T_2 = 300(6)^{0.286} = 501 \text{ K}$$

The specific work of the compressor is: $-w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.0 \text{ kJ/kgK}(501 - 300)\text{K}$

$$-w_C = 202 \text{ kJ/kg}$$

The first turbine work equals the compressor work:

$$w_{T1} = -w_C = 202 = C_{P0}(T_4 - T_5) = 1.0(1600 - T_5) \text{ find: } T_5 = 1399 \text{ K}$$

The intermediate pressure P_5 is found by:

$$s_5 = s_4 \Rightarrow P_5 = P_4(T_5/T_4)^{k/(k-1)} = 600(1399/1600)^{3.5}$$

$$P_5 = 375 \text{ kPa}$$

(b) The drive turbine is an isentropic process: $s_6 = s_5 \Rightarrow T_6 = T_5(P_6/P_5)^{(k-1)/k}$

$$T_6 = 1399(100/375)^{0.286} = 959 \text{ K}$$

The second turbine gives the specific net work output of the engine

$$w_{T2} = C_{P0}(T_5 - T_6) = 1.0(1399 - 959) = 442 \text{ kJ/kg}$$

(c) The mass flow rate is found from the stated power output:

$$\mathbf{m} = W_{\text{NET}}/w_{T2} = (150\text{kJ/s})/442 \text{ kJ/kg} = \mathbf{0.339 \text{ kg/s}}$$

(d) The air temperature entering the burner equals the exit temperature of the power turbine in an ideal regenerator $\Rightarrow T_3 = T_6 = \mathbf{959 \text{ K}}$

(e) The thermal efficiency of the cycle is computed by: $\eta_{\text{TH}} = w_{\text{NET}}/q_{\text{H}}$

$$q_{\text{H}} = C_{p0}(T_4 - T_3) = 1.0(1600 - 959) = 644 \text{ kJ/kg}$$

$$\eta_{\text{TH}} = 442/644 = \mathbf{0.687}$$