ME 18b, HW 5

Due Thursday May 14, 2009 (accepted until 4 pm)

Note: This homework is due on a Thursday not a Tuesday

TAs for HW #5:
Dan Alvarez, Sunday 8-10 pm, SFL 229;
Mary Dorman, Monday 7-9 pm, SFL 331

Number of classes attended: ______

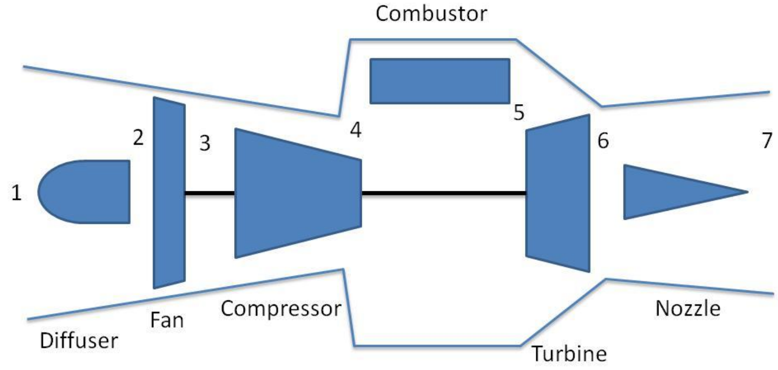
Hours spent on homework: _______

1. Brayton Cycle Jet Aircraft Engine The General Electric GE90 is a family of high-bypass turbofan engines built by GE-Aviation for the Boeing 777, with thrust ranging from 74,000 to 115,000 lbf (329 to 512 kN). It was first introduced in November 1995. The GE90 series are physically the largest engines in aviation history. The fan diameter of the latest variant, the GE90-115B, has a diameter of 325 cm (128 in). This means that the GE90 has a diameter larger than most cabins in business aircraft. For more info on this engine go to: http://www.turbokart.com/about_ge90.htm

Using the state numbers shown in the figure below for your analysis; determine the following operating conditions of the engine as power is applied prior to take-off while the plane is still stationary on the runway:

a. Air mass flow rate for a thrust of 400 kN.
b. Air volumetric flow rate in m^3/min and cfm.
c. The work of the fan, compressor and turbine. The fan has a pressure ratio of 2.1 and the compressor has a pressure ratio of 19.0. Ignore the portion of fan work that provides bypass air (which is not shown).
d. The heat input for final combustion temperature of T_5 = 1400 K.
e. The exhaust velocity
f. The fuel consumption in kg/hr if the fuel has a heating energy value of 45 MJ/kg.

You will need to make your own assumptions about the inlet conditions.



Solution:

Assume inlet conditions are 100 kPa and 295K and Vin = 0. A reasonable pressure range would be from about 75 kPa to 102 kPa for a ground level of 7500 ft to -200 ft above sea level. A reasonable temperature range would be from -40° to +40°C (cold Canadian winters to hot desert summers).

State 1: T1 = 22°C, 295K, h1 = 296 kJ.kg, Pr1 = 1.053

State 2: normally an isentropic compression process when the vehicle is moving, but since it is stationary, there are no changes in properties from state 1.

State 3: Isentropic compression by the fan with a pressure ratio of 2.1.
Pr3 = Pr2(P3/P2) = 1.053(2.1) = 2.21so we find from air properties table: T3 = 92C or 365 K, and h3 = 366 kJ/kg. The fan specific work is: wfan = h3 – h2 = 70 kJ/kg. P3 = 2.1(P2) = 210 kPa.

State 4: Isentropic compression by the compressor with a pressure ratio of 19.0
Pr4 = Pr3(P4/P3) = 2.21(19.0) = 41.99 so we find from air properties table: T4 = 546C or 819 K, and h4 = 843 kJ/kg. The compressor specific work is: wcomp = h4 – h3 = 477 kJ/kg. P4 = 19(P3) = 3.99 MPa

State 5: Isobaric combustion process with T5 = 1400 K or 1127C, we find from air properties table: h5 = 1515 kJ/kg and Pr5 = 361.8. The combustion specific heat input is: qin = h5 – h4 = 1515 – 843 = 672 kJ/kg. P5 = P4.

State 6: Isentropic expansion in the turbine. Assume the turbine work equals the compressor and fan work, wturb = wfan + wcomp = 70 + 477 = 547 kJ/kg. Also, wturb = h5 – h6 so
h6 = h5 – wturb = 1515 – 547 = 968 kJ/kg. From air table find: T6 = 658C or 931K and
Pr6 = 69.1. Find P6 = P5(Pr6/Pr5) = (3990 kPa)(69.1/361.8) = 762 kPa.

State 7: Isentropic expansion in the nozzle. Pr7 = Pr6(P7/P6) = 69.1(100/762) = 9.07, so we find from the air properties table: T7 = 269C or 542 K and h7 = 547 kJ/kg.

Since there is no net work from the jet engine, the difference between the heat input and the heat lost to the environment is the change in kinetic energy of the air. From the change in kinetic energy we can compute the exhaust velocity.

qout = h7 – h1 = 547 – 296 = 251 kJ/kg.

Exhaust velocity: ½$(Vout)^2$/1000 = qin – qout = 672 – 251 = 421 kJ/kg, so **Vout = 918 m/s.** (in reality this exceeds the speed of sound so we're ignoring compressibility effects.)

The thrust determines the mass flow rate through the engine: T = m(Vout – Vin)
m = (400,000N)/(918m/s). **m = 436 kg/s**.

Now that we know the air mass flow rate, we can compute the work of the fan, compressor and turbine:
Wfan = (m)wfan = (436 kg/s)(70 kJ/kg)/(1000MW/kW). **Wfan = 30.5 MW**
Wcomp = (m)wcomp = (436 kg/s)(477 kJ/kg)/(1000MW/kW). **Wcomp = 208 MW**

Wturb = 208 + 30.5. **Wturb = 238 MW**.

The heat input for combustion is:
Qin = (m)qin = (436 kg/s)(672 kJ/kg)/(1000MW/kW). **Qin = 293 MW**

The fuel consumption rate is mfuel = Qin/hfuel = (293 MW)(3600sec/hr)/(45 MJ/kg)
mfuel = 23,400 kg/hr

Volumetric flow rate: V = m/ρ = (436 kg/s)/(1 kg/m^3)(60s/min). **V = 26200 m^3/sec**
or in English units: V = (35.3 ft^3/m^3)(26200 m^3/sec). **V = 916,000 cfm**.

That's an impressive air and fuel flow rate! The fuel flow rate calculated here is 2x the actual flow rate because we've assumed all the thrust comes from the hot exhaust gases, we've ignored the contribution from the bypass fan.

2. (12.68) Otto Cycle Engine: A 3.3 L gasoline engine runs at 2400 RPM with a compression ratio of 10:1. The intake is at 95 kPa, 7°C. The heat addition to the engine during combustion is 900 kJ/kg. Determine the following using a method that account for variation in specific heat:

- **a.** The temperature and pressure at each point in the cycle.
- **b.** The cycle thermal efficiency.
- **c.** The power of the engine in kW.

Solution:

(a) State 1, T1 = 7°C or 280K, P1 = 95 kPa, from air table find u1 = 200 kJ/kg and Vr1 = 213

we will need the specific volume later for computing the Mean Effective Pressure. v1 = RT1/P1

v1 = (0.287 kPa m3/kg K)(280 K)/(95 kPa) = 0.846 m3/kg

State 2, Isentropic compression Vr2 = Vr1(V2/V1) = 213(1/10) = 21.3 From air table find:

T2 = 413°C or 686 K and u2 = 502 kJ/kg.

Find P2 using ideal gas law: P2 = P1(T2/T1)(V1/V2) = 95 kPa(686/280)(10). **P2 = 2328 kPa**.

State 3, Isochoric heat addition: qin = u3 – u2, so u3 = 900 + 502 = 1402 kJ/kg. From air table find: **T3 = 1438°C** or 1711 K and Vr3 = 1.35

Find P3 using ideal gas law: P3 = P2(T3/T2)(V2/V3) = 2328 kPa(1711/686)(1). **P3 = 5806 kPa**.

State 4, Isentropic expansion, Vr4 = Vr3(V4/V3) = 1.35(10) = 13.5. Find from air table: **T4 = 535°C** or 808 K and u4 = 599 kJ/kg.

Find P4 using ideal gas law: P4 = P3(T4/T3)(V3/V4) = 5806 kPa(808/1438)(1/10). **P4 = 326 kPa**.

(b) qout = u4 – u1 = 599 – 200 = 399 kJ/kg

wnet = qin – qout = 900 – 399 = 501 kJ/kg

η_{th} = wnet/qin = 501/900. **η_{th} = 56%**

(c) Mean Effective Pressure: MEP = wnet/v1(1 – 1/r), MEP = (501 kJ/kg)/[(0.846 m3/kg)(1 – 1/10)]

MEP = 658 kPa

W = MEP(Vmax – Vmin) = (658 kPa)(0.003 m3) = 1.97 kJ/cycle

Power = W*RPM/2 = (1.97 kJ/cycle)(2400 RPM)(min/60s)/2 = **39.5 kW = 53 hp**

3. (12.86 and 12.87) Diesel Cycle Engine: A 4-stroke diesel engine has a bore of 0.1 m, a stroke of 0.11 m. The compression ratio is 19:1. The temperature at the end of combustion is 2400 K. State your own assumptions for inlet conditions. Determine the following using cold air assumptions:

- **a.** The pressure and temperature at each point in the cycle.
- **b.** The cycle thermal efficiency.
- **c.** The engine power in kW and horsepower, hp. for a 6 cylinder engine running at 2000 RPM.

Solution:

Assume inlet conditions for State 1 are **T1 = 25C (298 K)** and **P1 = 100 kPa**

State 2: Isentropic compression from 1 to 2.

$T2 = T1(V1/V2)^{k-1} = 298K(19)^{0.4}$ **T2 = 968 K**

$P2 = P1(V1/V2)^{k} = 100\ kPa(19)^{1.4}$ **P2 = 6170 kPa**

State 3: Isobaric heat addition to gas from 2 to 3

P3 = P2

T3 = 2400 K, V3/V2 = T3/T2 = 2400/968 = 2.48 (cutoff ratio)

State 4: Isentropic expansion

$T4 = T3(V3/V4)^{k-1}$, V3/V4 = (V3/V2)(V2/V1) = 2.48/19 = 0.131

$T4 = 2400(0.131)^{0.4}$ **T4 = 1063K**

$P4 = P3(V3/V4)^{k} = 6170\ kPa(0.131)^{1.4}$ **P4 = 358 kPa**

Cycle Thermal Efficiency η_{th} = wnet/qin where wnet = qin – qout

qin = Cp(T3 – T2) = (1.0 kJ/kgK)(2400 – 968)K = 1432 kJ/kg

qout = Cv(T4 – T1) = (0.713 kJ/kgK)(1063 – 298) = 765 kJ/kg

wnet = 1432 – 765 = 667 kJ/kg

η_{th} = 667/1432; **η_{th} = 47%**

Engine Power, find air mass flow rate

The displacement is $\Delta V = 6 \times \pi(\text{Bore}/2)^2 \times \text{Stroke} = 6 \times \pi(0.1/2)^2 \times 0.11 = 0.0052\ m^3$

air mass flow rate is found using ideal gas law:

m = P1V1/RT1 = (100 kPa)(0.0052 m^3)/0.287 kJ/kgK)(298 K) = 0.0061 kg/cycle

Power = wnet x m x RPM/2 = (667 kJ/kg)(0.0061 kg/cycle)(2000/2 cycles/min)(min/60s)

Power = 67 kW or 90 hp.