ME 18b, HW 6

Due Thursday May 21, 2009 (accepted until 4 pm)

Note: This homework is due on a Thursday not a Tuesday

TAs for HW #6:
Xiaobai and Julianne

Number of classes attended: ______

Hours spent on homework: _______

1. Ideal Gas Mixtures A mixture of 60% nitrogen and 40% oxygen on a molar basis is contained in a frictionless piston-cylinder assembly at 200 kPa and 175°C. The cylinder has an initial volume of 0.8 m^3. The mixture is compressed adiabatically until the pressure reaches 450 kPa. Determine the final temperature of the mixture and the work for this process. Compare these values to air using either the cold air standard assumptions or variable specific heat method for the process.

Solution:

Energy Eqn. for adiabatic compression process: $Q_{12} – W_{12} = m(u_2 – u_1)$

Since $Q_{12} = 0$, then $W_{12} = mCv(T_2 – T_1)$

and $T_2 = T_1(P_2/P_1)^{(k-1)/k}$

where values for m, k, and Cv are based on mixture composition.

working on a mass basis: $c_{N2} = y_{N2}M_{N2}/(y_{N2}M_{N2} + y_{O2}M_{O2}) = 0.6(28)/(0.6(28) + 0.4(32))$

then $c_{N2} = 0.57$ and $c_{N2} = 0.43$

$k_{mix} = Cp_{mix}/(Cp_{mix} – R_{mix})$

$Cp_{mix} = c_{N2}Cp_{N2} + c_{O2}Cp_{O2} = 0.57(1.042\ kJ/kgK) + 0.43(0.922\ kJ/kgK) = 0.990\ kJ/kgK$

$R_{mix} = c_{N2}(R/M_{N2}) + c_{O2}(R/M_{O2}) = 0.57(8.314/28) + 0.43(8.314/32) = 0.281\ kJ/kgK$

$Cv_{mix} = Cp_{mix} - R_{mix} = (0.990 – 0.281) = 0.709\ kJ/kgK$

$k_{mix} = 0.990/0.709 = 1.396$

$T_2 = 448K(450/200)^{0.396/1.396}$ **T_2 = 564 K or 291°C**

$m = PV/RT = (200\ kPa·0.8\ m^3)/(0.281\ kPa\ m^3/kgK·448K) = 1.27\ kg$

$W_{12} = (1.27\ kg)(0.709\ kJ/kgK)(564 – 448)K$; **$W_{12}$ = 104 kJ**

Isentropic compression process, cold-air std assumption: $T_2 = T_1(P_2/P_1)^{(k-1)/k}$

$T_2 = 448\text{K}(450/200)^{(0.4/1.4)}$ = **565 K or 292°C**

$W_{12} = m(u_2 - u_1) = m \cdot Cv(T_2 - T_1)$

$m = PV/RT = (200\ \text{kPa} \cdot 0.8\ \text{m}^3)/(0.287\text{kPa m}^3/\text{kgK} \cdot 448\text{K}) = 1.24\ \text{kg}$

$W_{12} = (1.24\ \text{kg}) \cdot (0.713\ \text{kJ/kgK}) \cdot (565 - 448)\text{K}$; $\mathbf{W_{12} = 104\ kJ}$

Isentropic compression process, variable specific heat analysis:

State 1: $T_1 = 175°\text{C}$, $u_1 = 322$ kJ/kg, $Pr_1 = 4.578$

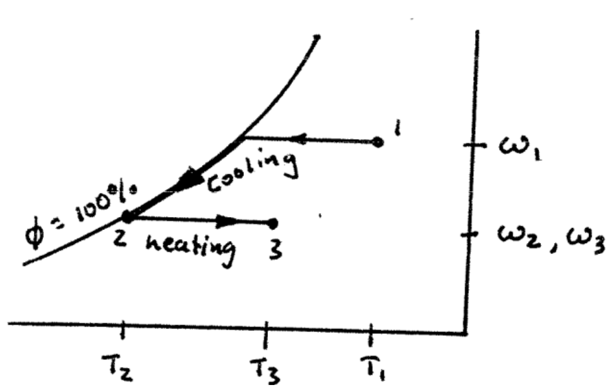
State 2: $Pr_2 = Pr_1(P_2/P_1) = 4.578(450/200) = 10.30$, find that $\mathbf{T_2 = 289°C}$ and $u_2 = 406$ kJ/kg

$W_{12} = (1.24\ \text{kg}) \cdot (406 - 322)$; $\mathbf{W_{12} = 105\ kJ}$

Comments: In this case the cold-air standard values are very close to those obtained with variable specific heat. The composition of the given gas mixture isn't very different from ordinary air. As a result, there is not a significant difference in the final temperature or the amount of work in the process.

2. Air Conditioning Process A combination cooling and reheat process is used to deliver air at a dry bulb temperature of 20°C and a relative humidity of 40%. The air enters at a dry bulb temperature of 29°C with a relative humidity of 70% and a volumetric flow rate of 45 m^3/min. Determine the heat transfer rate in the cooling section and the heating section and the mass flow rate of the condensate from the cooling section.

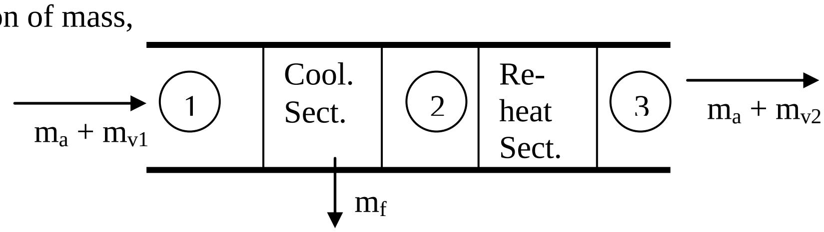
Solution:



In the cooling section: Conservation of mass,

dry air: $m_{a1} = m_{a2} = m_a$.

water: $m_{v1} = m_f + m_{v2}$



or: $\omega_1 m_a = m_f + \omega_2 m_a$ or: $m_f = (\omega_1 - \omega_2)m_a$

The mass flow rate of air is: $m_a = P_{a1}V_1/RT_1$

$P_{a1} = P – P_{v1}$, at 29°C the saturation pressure of water is P_{g1} = 4.0 kPa,

so the vapor pressure is: P_{v1}, = $\phi \cdot P_{g1}$ = 0.70(4.0 kPa) = 2.8 kPa

P_{a1} = 100 – 2.8 = 97.2 kPa

m_a = (97.2 kPa)(45 m^3/min)/(0.287 kPa m^3/kgK)(302K) = 50.5 kg/min

The inlet humidity ratio is ω_1 = 0.0177 kg water/kg dry air.

The humidity ratio at state 2 is equal to that of state 3, ω_2 = 0.0058 kg water/kg dry air.

The condensate mass flow rate is: m_f = (0.0177 – 0.0058)(50.5 kg/min)

m_f = 0.60 kg H20/min

Conservation of energy in cooling section: $Q_{12} = m_a(h_2 – h_1) + m_f h_f$

where h_1 = 94.42 kJ/kg d.a. and h_2 is found using RH = 100% and ω_2 = 0.0058

h_2 = 40.68 kJ/kg d.a. and T_2 = 6°C dry bulb temp. Then from steam table find h_f = 25.2 kJ/kg for sat liq. at 6 °C.

The heat transfer rate in the cooling section is:

Q_{12} = (50.5 kg/min)(40.68 – 94.42) kJ/kg d.a. + (0.60 kg H20/min)(25.2 kJ/kg)

Q_{12} = -2700 kJ/min = -45.0 kW.

Conservation of mass in the heating section

dry air: $m_{a2} = m_{a3} = m_a$.

water: $m_{v2} = m_{v3} = m_v$

Conservation of energy: $Q_{23} = m_a(h_3 – h_2)$

for T_3 = 20°C and 40% RH, h_3 = 54.85 kJ/kg d.a.

Q_{23} = (50.5 kg/min)(54.85 - 40.68) kJ/kg d.a.

Q_{23} = 716 kJ/min = 11.9 kW

Comments: The cooling heat transfer is (-) because heat is removed from the air and the reheat is (+) because heat is added to the air.

3. Air Conditioning Process A 1000-m thick cloud (assume the ambient pressure is 100 kPa, and ignoring the effects of altitude) contains air at 27°C and 90% relative humidity. Suppose the cloud rises up and its temperature decreases by 20°C. Estimate the depth of rainfall produced by the cloud.

Solution:

From psychrometric chart at T_1 = 27°C and 90% RH: ω_1= 0.0204 kg water/kg d.a.

At T_2 = 7°C and 100% RH: ω_2= 0.0062 kg water/kg d.a.

Conservation of mass: $m_{rain} = (\omega_1 - \omega_2)m_a$

$m_a = P_{a1}V_1/RT_1$

$P_{a1} = P - P_{v1}$, at 27°C the saturation pressure of water is P_{g1} = 3.6 kPa,

so the vapor pressure is: P_{v1}, = $\phi \cdot P_{g1}$ = 0.90(3.6 kPa) = 3.2 kPa

P_{a1} = 100 – 3.2 = 96.6 kPa

m_a = (96.6 kPa)(1000 m)/(0.287 kPa m^3/kgK)(300K) = 1120 kg/m^2

m_{rain} = (0.0204 – 0.0062 kg water/kg d.a.)(1120 kg/m^2) = 15.9 kg/m^2

density of water: 1000 kg/m^3,

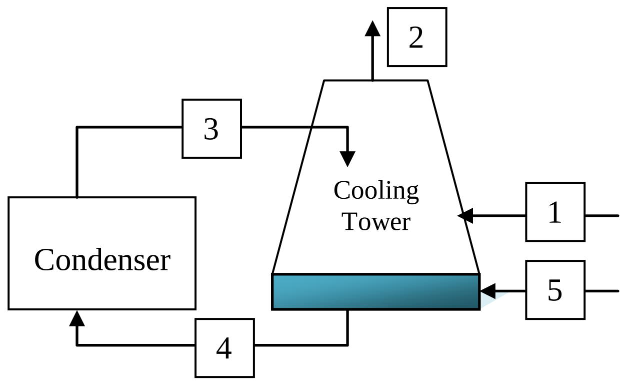
Depth of rain = (15.9 kg/m^2)/(1000 kg/m^3) = **1.6 cm**

4. Air Conditioning Process A power plant utilizes a steady-flow natural-draft, wet-cooling tower at 1 atmosphere. The air temperature entering the tower is 20°C and 50% RH and the air leaving the cooling tower is 32°C and 80% RH. Makeup water is available from a source that's at 15°C. Water from the condenser enters the cooling tower at 70°C at a flow rate of 100 kg/s and is cooled to 40°C. Calculate the following:

- **a.** The heat transfer rate from the condenser water
- **b.** The mass flow rate of makeup water required
- **c.** The volume flow rate of moist air entering the cooling tower
- **d.** The volume flow rate of moist air that leaves the cooling tower
- **e.** The humidity ratio of the air leaving the cooling tower
- **f.** The dew point temperature of the air leaving the cooling tower.

Solution:

1 is air in
2 is air out
3 is condenser water in
4 is condenser water out
5 is make up water



Summary of Properties

State	T, °C	RH, %	ω kg H2O/ kg d.a.	Psat, kPa	Pvap, kPa	h, kJ/kg	source
1	20	50	0.0073	2.3	1.2	58.75	Psych chart
2	32	80	0.0243	4.8	3.8	114.3	Psych chart
3	70	-	-	-	-	293	Sat. liq.
4	40	-	-	-	-	167.5	Sat. liq.
5	15	-	-	-	-	63.0	Sat. liq.

a) Condenser water is liquid in and out: $m_3 = m_4 = m_w$

$Q_{cond} = m_w(h_4 - h_3)$ = (100 kg/s)(167.5 – 293) kJ/kg; $\mathbf{Q_{cond} = -12{,}600\ kW}$

Conservation of mass for water: $m_5 = m_{v2} - m_{v1}$

Conservation of mass for air: $m_{a1} = m_{a2} = m_a$ and $m_5 = (\omega_2 - \omega_1)m_a$

Conservation of energy: Qin = Qout so $h_1 \cdot m_a + m_3 \cdot h_3 + m_5 \cdot h_5 = m_a \cdot h_2 + m_4 \cdot h_4$

$m_a = m_w(h_4 – h_3)/(h_1 – h_2 + (\omega_2 – \omega_1)h_5)$

The required air mass flow rate is:

m_a = 100 kg/s(167.5 – 293)/(58.75 – 114.3 + (0.0243 – 0.0073)63.0) = 230 kg/sec

b) The make-up water flow rate is: m_5 = 230 kg/sec((0.0243 – 0.0073); **m_5 = 3.9 kg/sec**

c) Find volumetric air flow rate from ideal gas equation $V = m_aRT/P_a$

$P_{a1} = P – P_{v1}$ = 100 -1.2 = 98.8 kPa

Inlet air volumetric flow rate: V_{a1} = (230 kg/sec)(0.287)(293K)/(98.8 kPa); **V_{a1} = 196 m^3/sec**

d) Outlet air volumetric flow rate:

$P_{a2} = P – P_{v2}$ = 100 - 3.8 = 96.2 kPa

Outlet air volumetric flow rate: V_{a2} = (230 kg/sec)(0.287)(305K)/(96.2 kPa); **V_{a2} = 209 m^3/sec**

e) Outlet humidity ratio: **ω2 = 0.0243 kg H20/kg d.a.**

f) Dew Point Temperature at outlet: **28.1°C** from psychrometric chart of CATT3.