ME 18b Homework Solutions

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Homework 2

Due Thursday April 15, 2008 (accepted until 5 pm).
Reading Sonntag, Borgnakke & Van Wylen, Ch. 11. For more information: *Gas Turbine Theory*, by Cohen, Rogers & Saravanamuttoo, which is in Sherman Fairchild Library.

2.1 Optimum temperature for reheating

As described in class, reheat increases the work output of a gas turbine. What is the optimum intermediate pressure at which to reheat gas?

To answer the question, consider a cycle that operates between a maximum pressure P_2 and a minimum pressure P_1. The maximum temperature of the cycle is T_{max}. The reheat occurs at a pressure P_{rh}, and the gas is reheated to the maximum temperature. Assume that the specific heats are constant and that the turbine is ideal. First sketch a reheat cycle for a turbine expansion on T–s coordinates and show the expansion for several different values for P_{rh}. Then determine a relation for the total work out of the turbine, and find an expression for P_{rh} in terms of P_1 and P_2 at which the work output of the turbine is a maximum.

Solution (10 points)

The total work can be expressed as

$$w_t = c_p\left(T_{\max} - T_a\right) + c_p\left(T_b - T_{\text{out}}\right).$$

The intermediate temperatures T_a and T_b can be expressed in terms of $T_{\max}$ and the pressure ratios:

$$\frac{T_a}{T_{\max}} = \left(\frac{P_a}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_{rh}}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \qquad \Rightarrow \qquad T_a = T_{\max}\left(\frac{P_{rh}}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_b}{T_{\text{out}}} = \frac{T_{\max}}{T_{\text{out}}} = \left(\frac{P_b}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_{rh}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \Rightarrow \qquad T_b = T_{\max}; \qquad T_{\text{out}} = T_{\max}\left(\frac{P_1}{P_{rh}}\right)^{\frac{\gamma-1}{\gamma}}.$$

The work output of the turbine can hence be written as

$$\underline{\underline{w_t = c_p T_{\max}\left[1 - \left(\frac{P_{rh}}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right] + c_p T_{\max}\left[1 - \left(\frac{P_1}{P_{rh}}\right)^{\frac{\gamma-1}{\gamma}}\right]}}.$$

In order to maximize the work output w_t of the turbine as a function of P_{rh} :

$$\frac{\partial w_t}{\partial P_{rh}} = -\frac{c_p T_{\max}}{P_2}\frac{\gamma-1}{\gamma}\left(\frac{P_{rh}}{P_2}\right)^{-\frac{1}{\gamma}} + \frac{c_p T_{\max}}{P_1}\frac{\gamma-1}{\gamma}\left(\frac{P_1}{P_{rh}}\right)^{2-\frac{1}{\gamma}} = 0$$

$$(P_{rh})^{\frac{2}{\gamma}-2} = (P_1)^{\frac{1}{\gamma}-1}(P_2)^{\frac{1}{\gamma}-1} \qquad \Rightarrow \qquad \underline{\underline{P_{rh} = \sqrt{P_1 P_2}}}.$$

Substituting back into the expression for the work out of the turbine we get

$$w_t = c_p T_{\max}\left[1-\left(\frac{\sqrt{P_1P_2}}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right] + c_p T_{\max}\left[1-\left(\frac{P_1}{\sqrt{P_1P_2}}\right)^{\frac{\gamma-1}{\gamma}}\right] = \underline{\underline{w_t = 2c_p T_{\max}\left[1-\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{2\gamma}}\right]}}.$$

2.2 Automotive gas turbine (Modified problem 11.75 in S, B & V.)

Gas turbines are primarily used for propulsion and power generation. However, other uses have been suggested. The book describes an automotive use for a gas turbine engine. This use is a rarity for several reasons. First, gas turbines have a slow torque response to movement of the accelerator pedal and are not able to cope with hills if the engine is geared for level running. In addition, considerable reduction gearing is needed between the high turbine rotational speed and the wheel speed. Regeneration is also essential for reasonable fuel consumption.

The cycle in the book is shown below. In the first turbine, the gas expands to pressure P_5, just low enough for this turbine to drive the compressor. The gas is then expanded through a second turbine connected to the drive wheels. Assume air as the working fluid and the specific heats are constant. The compressor has a *polytropic* efficiency of $\eta_{\infty c} = 0.80$, and the turbines have a *polytropic* efficiency of $\eta_{\infty t} = 0.85$. The regenerator has an efficiency of 0.75. There is a pressure drop in the burner and on both sides of the regenerator of 2 percent of the inlet pressure to the specific component. First sketch the cycle on T–s coordinates labeling all of the important isobars. Then find:

1. The intermediate pressure P_5.
2. The net work output of the engine per kilogram of air, and the mass flow rate through the engine.
3. The air temperature entering the burner T_3, and the thermal efficiency of the engine.

Solution (15 points)

The temperatures T_1 and T_4 are given. The pressures P_1, P_2, P_3, P_4, P_6, and P_7 are either given or can be obtained directly from known quantities

$$P_2 = 6P_1, \quad P_3 = 0.98P_2, \quad P_4 = 0.98P_3, \quad P_6 = P_7/0.98.$$

From the definition of polytropic efficiency, we can calculate the temperature after the compressor,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}\frac{1}{\eta_{\infty c}}} = 6^{\frac{0.4}{1.4}\frac{1}{0.8}} = 1.896 \qquad \rightarrow \qquad T_2 = 568.9K.$$

The first turbine stage provides the power to drive the compressor

$$w_t = c_p(T_4 - T_5) = c_p(T_2 - T_1) = w_c \quad \Rightarrow \quad T_5 = T_4 - (T_2 - T_1) = 1331.1k.$$

The pressure after the first turbine stage is

$$\frac{T_4}{T_5} = \left(\frac{P_4}{P_5}\right)^{\frac{\gamma-1}{\gamma}\eta_{\infty t}}; \qquad \frac{1600}{1331.1} = \left(\frac{576}{P_5}\right)^{\frac{0.4}{1.4}(0.85)} \quad \Rightarrow \quad \underline{\underline{P_5 = 270.1\,\text{kPa}}}.$$

Similarly, the temperature after the power turbine stage can be calculated

$$\frac{T_5}{T_6} = \left(\frac{P_5}{P_6}\right)^{\frac{\gamma-1}{\gamma}\eta_{\infty t}}; \qquad \frac{1331.1}{T_6} = \left(\frac{270.1}{102}\right)^{\frac{0.4}{1.4}(0.85)} \quad \Rightarrow \quad T_6 = 1050.75\,\text{K}.$$

The specific work of the power turbine is

$$w_{pt} = c_p\,(T_5 - T_6) = \underline{\underline{w_{pt} = 280.35\,\text{kJ}\,\text{kg}^{-1}}}.$$

The mass flow rate is obtained from the specific work of the power turbine and the given power output of 150 kW,

$$\dot{m} = \frac{\dot{W}_{\text{tot}}}{w_{pt}} = \frac{150}{280.35} = \underline{\underline{\dot{m} = 0.535\,\text{kg}\,\text{s}^{-1}}}.$$

To obtain the temperature of the air entering the burner, T_3, we need to use the definition of the efficiency of the regenerator,

$$\eta_{\text{regen}} = \frac{Q_{actual}}{Q_{maximum}} = \frac{h_3 - h_2}{h_6 - h_2}.$$

The maximum heat transfer is such that the fluid being heated reaches the highest possible temperature at the end of the regenerator. The highest possible temperature is equal to the highest temperature of the cooling side, i.e., T_6.

$$\eta_{\text{regen}} = \frac{T_3 - T_2}{T_6 - T_2}; \qquad 0.75 = \frac{T_3 - 568.9}{1050.75 - 568.9} \quad \Rightarrow \quad \underline{\underline{T_3 = 930.29\,\text{K}}}.$$

The thermal efficiency of the engine is

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{h_5 - h_6}{h_4 - h_3} = \frac{T_5 - T_6}{T_4 - T_3} = \frac{1331.1 - 1050.75}{1600 - 930.29} = \underline{\underline{\eta_{\text{th}} = 0.4186}}.$$

2.3 The X–1 (experimental supersonic rocket aircraft)

In the late 1940's the US Air Force with Bell Aircraft Company developed a series of experimental planes that utilized a rocket engine for propulsion. The first designs were launched in flight from a B–29 mother-ship. On Jan. 5, 1949, the X–1 made its first rocket-propelled take-off under the command of Captain Chuck Yaeger. The X–1 reached Mach 1.4 at an altitude of 23 000 ft (7015 m) in 1 minute, 40 seconds. The total flight lasted 8 minutes (approximately 6 minutes of glide time).

The flight was relatively short because rockets do not use the surrounding air as an oxidizer. Instead a rocket carries both the fuel and the oxidizer, which are pressurized using an internal pump or pressurizing system. The fuel and the oxidizer are injected into the combustion chamber and the high temperature combustion products are ejected through the nozzle.

According to the *Rocket Encyclopedia Illustrated* (ed. Herrick, Aero Publishers, Inc., 1959), the fuel of X–1 was a 25–75% mixture of water and alcohol. Liquid oxygen is also carried. “The igniter–chamber assembly was started by spraying a stream of fuel and a stream of gaseous oxygen in a region of a spark plug. Chamber pressure was approximately 230 psi (1.57 MPa), with an accompanying temperature of 5000°R

(2780 K)." On the X–1, there were 4 rockets, each providing 1500 lb thrust (6.67 kN thrust). The individual chambers could be operated independently or in combination, which was the only means of thrust control.

We are interested in computing the engine parameters for the X–1 using the combustion chamber pressure of 1.57 MPa, and a temperature of 2780 K. Assume that the combustion gases behave as a "frozen" ideal gas mixture (the term "frozen" is used to designate a mixture that is not reacting). The mixture has a molecular weight of 25 and a ratio of specific heats of $\gamma = 1.2$; from these values you can calculate the specific heat at constant pressure. Assume that the specific heat is constant.

a) For static conditions (on the ground), what is the exit velocity relative to the rocket, V_j, and the exhaust Mach number? Assume that the exhaust pressure is 100 kPa.

b) What is the mass flow rate for a single rocket for a static thrust of 6.6 kN?

c) At an altitude of 7000 m, the atmospheric temperature is 239.5 K and the pressure is 40.2 kPa. What are the exit velocity relative to the rocket, the exhaust Mach number, and the mass flow rate for a total thrust of 26.7 kN at flight altitude?

d) What is the flight speed at a Mach number of 1.4 at an altitude of 7000 m?

Solution (20 points)

For an ideal gas, the specific heats are related by $c_p - c_v = R$ and $c_p / c_v = \gamma$. Therefore,

$$c_p = \frac{\gamma}{\gamma - 1} R = 49.89\,\mathrm{kJ\,kmol^{-1}\,K^{-1}} = 1.996\,\mathrm{kJ\,kg^{-1}\,K^{-1}}.$$

For an isentropic (ideal) expansion of the gas, we have

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}; \qquad \frac{T_2}{2780} = \left(\frac{0.1}{1.57}\right)^{\frac{0.2}{1.2}} \qquad \rightarrow \qquad T_2 = 1756.82\,\mathrm{K}.$$

Using the first law for open systems (it should be noted that the units of h are $\mathrm{kJ\,kg^{-1}}$),

$$\left(h + \cancelto{0}{V^2}/2\right)_{in} = \left(h + V^2/2\right)_{out}$$

$$V_2 = \sqrt{2c_p(T_1 - T_2)} = \underline{\underline{2021\,\mathrm{m/s}}}$$

For an ideal gas, the Mach number can be calculated by:

$$M = \frac{V}{\sqrt{\gamma R T_2}} = \frac{2021}{\sqrt{1.2 \cdot \frac{8315}{25} \cdot 1756.82}} = \frac{2021}{837} = \underline{\underline{2.41}}$$

The static thrust is given by:

$$T = V_j \dot{m}, \qquad \rightarrow \qquad \dot{m} = \frac{T}{V_j} = \frac{6.6\,\mathrm{kN}}{2021\,\mathrm{m/s}} = \underline{\underline{3.27\,\mathrm{kg/s}}}$$

If the exit conditions are modified to $P_{out} = 40.2$ kPa and $T_{env} = 239.5$ K (note that the environment temperature is only used in the calculation of the Mach number of the airplane), the jet properties can be recalculated as follows:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}; \qquad \frac{T_2}{2780} = \left(\frac{0.0402}{1.57}\right)^{\frac{0.2}{1.2}} \qquad \rightarrow \qquad T_2 = 1509.27\,\mathrm{K}$$

$$V_2 = \sqrt{2c_p(T_1 - T_2)} = \underline{\underline{2252.3\,\mathrm{m/s}}}$$

$$M = \frac{V}{\sqrt{\gamma R T_2}} = \frac{2252.3}{\sqrt{1.2 \cdot \frac{8315}{25} \cdot 1509.27}} = \frac{2252.3}{776} = \underline{\underline{2.90}}$$

The static thrust is given by:

$$T = V_j \dot{m}, \quad \rightarrow \dot{m} = \frac{T}{V_j} = \frac{26.7\,\mathrm{kN}}{2252.3\,\mathrm{m/s}} = \underline{\underline{11.85\,\mathrm{kg/s}}}$$

The airspeed at T_{env} can be calculated by:

$$c = \sqrt{\gamma R T_{env}} = \sqrt{1.4 \cdot \frac{8315}{29} \cdot 239.5} = 310\,\mathrm{m/s}$$

Then, if the airplane is flying at Mach number of 1.4, the flight speed is:

$$M = \frac{V}{c} \quad \rightarrow V = M \cdot c = \underline{\underline{434.08\,\mathrm{m/s}}}$$