ME 18b, HW 4

Due April 29, 2008 by 5 pm

4.1 Stirling Cycle

a) Consider an ideal Stirling power cycle with perfect regeneration, as discussed in class. **Show** that the thermal efficiency of an ideal Stirling cycle is given as the following,

$$\eta_{th} = \frac{w_{net}}{q_H} = 1 - \frac{T_L}{T_H}$$

where T_L and T_H are the temperatures of the reservoirs.

Note that the efficiency is independent of the properties of the working fluid, and is equivalent to the thermal efficiency of a Carnot engine.

b) The U.S. Department of Energy through the Sun Lab (see http://www.energylan.sandia.gov/sunlab/overview.htm#dish) and various industrial partners is developing 25-kW Stirling engines. These engines are solar powered and use an array of mirrors that focus the sun's rays at a high-temperature receiver. The receiver contains heater tubes, through which the engine's working fluid passes and absorbs heat. To increase the energy transfer between the receiver and the working fluid, high-pressure helium or hydrogen is often used.

In this design of the Stirling engine, the maximum temperature is around 700^{o}C and a maximum pressure is 20 MPa. Assume that the minimum temperature is 20^{o}C, and that the minimum pressure is 0.2 MPa (it should remain above atmospheric pressure so that air doesn't leak into the system). Using helium as the working fluid and assuming that the cycle is ideal, what is the mass flow rate of helium through the engine to produce 25 kW of power?

c) The thermal efficiency is reportedly 42% with a solar-to-electric conversion efficiency of 23% at peak. Do you have any comments on these values as compared with the ideal value?

4.2 Caltech power plant.

In Caltech's power plant, a gas turbine cycle is used as a topping cycle to a Rankine cycle. The attached Fig. 1 shows an approximate layout of the plant with the appropriate numbering. Using a heat exchanger, the exhaust heat from the gas turbine provides some of the heat input needed to boil the water for the steam cycle. The rest of the heat input is provided by a separate boiler. (Caltech also produces steam for campus heating and other uses, but forget about this part of the power plant.)

The steam coming into the steam turbine is superheated vapor at T_8 and 2.5 MPa. The pressure of the exhaust from this turbine is 0.5 MPa (this pressure is fairly high because the steam is used on campus). The power output of this turbine is 2.5 MW, and the isentropic steam turbine efficiency is 80%. The pump efficiency is 70%.

The gas turbine and compressor have *polytropic* efficiencies of 85% and 80%, respectively. The compression ratio for the turbine and compressor is 16; the compressor inlet temperature is 300 K, and the turbine inlet temperature is 1600 K. Use the ratio of specific heats as $\gamma = 1.4$ and a constant $c_p = 1.0$ kJ/kgK.

For this problem, try using the CATT program and export the data to an excel spreadsheet.

a) First sketch both cycles on two different T-s coordinates with the same temperature scale and using the numbering from Fig. 1

b) First consider the gas turbine. Find the turbine outlet temperature, the turbine work output in kJ/kg, and the work input to the compressor in kJ/kg.

c) If the gas turbine cycle provides a **net** power of 10 MW, what is the gas flow rate?

d) Now consider the steam turbine. Assume that the steam is superheated to a temperature of $T_8 = 280^{o}C$. Find the quality of the water/vapor mixture the exits the steam turbine. Remember that the turbine is not ideal.

e) Calculate the mass flow rate of the steam. (Remember that the power output of the steam turbine is 2.5 MW.)

f) Calculate the work input to the pump.

g) Determine the rate of heat input to the Rankine cycle in kW.

h) Now consider the heat exchange process between the two cycles. Because of pollution considerations, the temperature T_5 must be at least $165^{o}C$. Using $T_5=165^{o}C$, What fraction of the heat needed to run the Rankine cycle is provided by the waste heat from the gas turbine? Note that the rest of the heat is provided from the boiler.

i) Determine the overall thermal efficiency for the entire Brayton/Rankine power plant. Be careful. (In the actual plant the steam that is exhausted from the turbine at 0.5 MPa is used for other things on campus, which would increase the overall efficiency).

j) Now, let's see if we can increase the efficiency by adjusting the temperature of T_8. Remember that the exit quality from the steam turbine should be between $0.9< x_8 <1.0$. Try a temperature higher and lower than $280^{o}C$ for T_8, while keeping the quality within the stated range. Can you increase the overall efficiency?

Problem 4.2

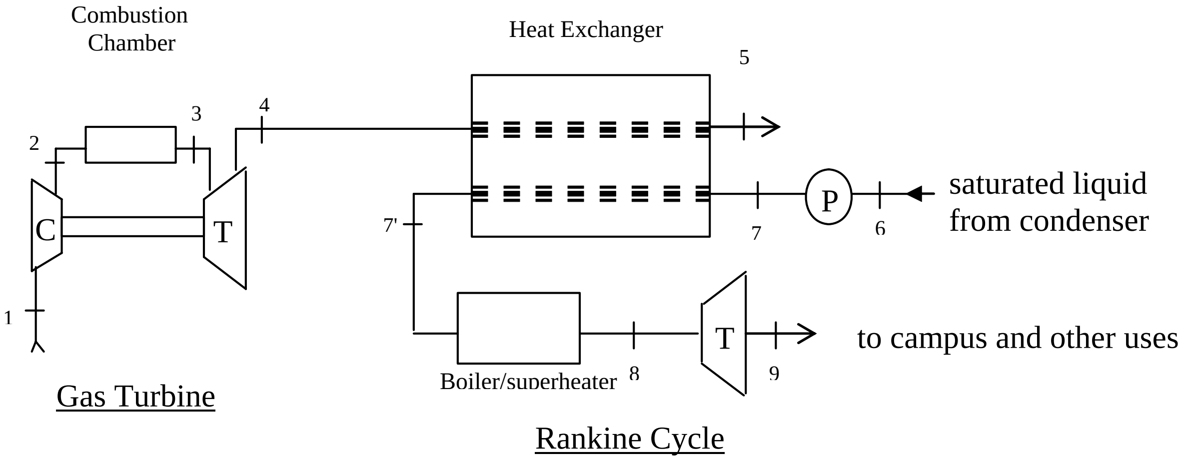

Figure 1. Combined Cycle

4.3 Reheat and regenerative heating

In this problem we want to examine a simplified version of the Pasadena power plant as shown in the figure. In the plant, the pressure into the turbine is 12.5 MPa and the temperature is 550ºC. The fluid is expanded in the first turbine and then reheated at a pressure of 4.0 MPa to a temperature of 550ºC. The fluid is then expanded in the second turbine. A single open feed water heater is used to improve the cycle thermal efficiency. The feed water heater is operated at a pressure of 1.0 MPa. The condenser pressure is 5 kPa. The isentropic turbine and pump efficiencies are 85% and 80%, respectively. The fluid at states 1 and 3 is a saturated liquid. States 2 and 3 are at a pressure of 1.0 MPa.

a) Draw the T-s diagram showing the vapor dome and the appropriate isobars.
b) What is the quality of the fluid exiting the second turbine?
c) What is the ratio of the mass flow rate of steam bled from the turbine for regeneration compared to the mass flow into the turbine?
d) If the net power output of the plant is 70 MW, what is the mass flow rate into the first turbine (be careful)? What is the back work ratio (power into pumps divided by power out of turbines)?
e) Calculate the plant thermal efficiency.
f) Suppose the plant is burning natural gas. Residential natural gas costs \$0.95 per therm. A therm is 100,000 BTU (British Thermal Unit) and 1 BTU = 1.055 kJ. If the combustion process has a conversion efficiency of 80% (not all of the fuel is completely burned in the combustion process) calculate the cost to generate 1 kW-hour of electricity. Compare this figure with the rate paid by So. Cal. Edison customers of \$0.15 per kW-hr.

Problem 4.3